International Journal of Theoretical Physics, Vol. 29, No. 8, 1990

Photon–Electron Scattering Taking Vacuum Energy into Account

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Received October 12, 1988

We deal with photon–electron scattering between the two uncharged conducting parallel plates. The electromagnetic vacuum field between the two plates is defined by the configuration of space and also interacts with the electrons. We first deal with the relevant operators for the electron and photon fields and with the Feynman propagator. We compute the S-matrix for photon–electron scattering, taking into account the influence of the vacuum field. The computed photon–electron scattering cross section also manifests the influence of the vacuum field. We give an example for low-energy scattering of the influence of the vacuum field upon the scattering cross section.

1. INTRODUCTION

Vacuum energy in quantum electrodynamics can be avoided by selecting any initial energy level higher than the vacuum energy whereupon we compute with the relative energy differences. The absolute energy level makes sense in the general theory of relativity, where energy creates curvature of space. Vacuum energy within the general theory of relativity constitutes the source of the basic curvature of space. The presence of vacuum energy can also be established with many other phenomena in physics. Experimental confirmation of the zero-point energy ideas may be found in the general consistency of quantum mechanics in describing actual microscopic phenomena, and also in certain experimental results which depend specifically on the quantum zero-point vibrations. Perhaps one of the most striking phenomena is the failure of liquid ^{4}He to solidify at normal pressures as the temperature is reduced toward the absolute zero (Finkelnburg, 1964). Bose condensation predicts that all helium atoms can fall into the same lowest energy state. However, the zero-point vibrations of this light atom are sufficient to overcome the attraction of the very small

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0020-7748/90/0800-0825$06.00/0

van der Waals forces, so that no crystal lattice is formed until considerable pressure is applied. Zero-point energy effects are also found in the vapor pressure of the neon isotopes (Casimir, 1951). Nuclear interactions and rotational moments in molecules require the inclusion of zero-point vibrations in their explanation (Ramsey, 1963). The finite random scattering of x-rays from crystals at very low temperatures shows the presence of zero-point oscillations of the molecules of the crystal (Peierls, 1955). Measurements of molecular specific heats and paramagnetism are in agreement with the presence of zero-point vibrations (Einstein and Stern, 1913).

Casimir established that the vacuum fluctuation of the electromagnetic field generates the attractive force of the two noncharged conducting parallel plates (Casimir, 1948; Fierz, 1960). When computing the vacuum energy, each term $\sum \frac{1}{2}\hbar\omega = E_i$ (ω depends on the configuration of the space) is formally divergent. Casimir solved the problem of divergence by a corresponding cutoff function $e^{-\lambda\omega/c}$. If the vacuum energy of the system for infinite separation is set equal to zero, then the energy per unit area of the plates for any finite separation L_z is $-\pi(720L_z^3)^{-1}$. The first verification of the Casimir effect was performed experimentally by Spaarnay (1958), who used one chromium plate and one chromium/steel plate with a surface of 1 cm^2 each. With a distance of 0.5 μm the attractive force between the plates was 0.2 dyne/cm^2. A very good result for the computation of vacuum energy for the conducting sphere was achieved by Boyer (1968).

This paper deals with the scattering of light by electrons located between two noncharged conducting parallel plates. Between the two plates there is an electromagnetic vacuum field which also interacts with the electrons. From the computed S-matrix and the scattering cross section it is evident that the vacuum field interacts with the Dirac field, as the contributions due to the interaction of the Dirac field with the electromagnetic vacuum field in the S-matrix and the cross section differ from zero.

2. DESCRIPTION OF THE PROBLEM

This paper deals with the scattering of photons by the electrons located between the two noncharged conducting parallel plates (Figure 1). As the field is spread all over the space, it is generally supposed that the field is enclosed in a large cube whose edges L_x, L_y, and L_z are parallel with the space coordinates. We also presume that each field variable simultaneously constitutes a periodic function of the space. Within the limit the lengths L_x, L_y, and L_z approach infinity. In our case the hypothesis is that only L_x and L_y approach infinity, whereas L_z constitutes an extremely minute span between the two plates ($L_z < \mu$m). It is this assumed Minkowski space that

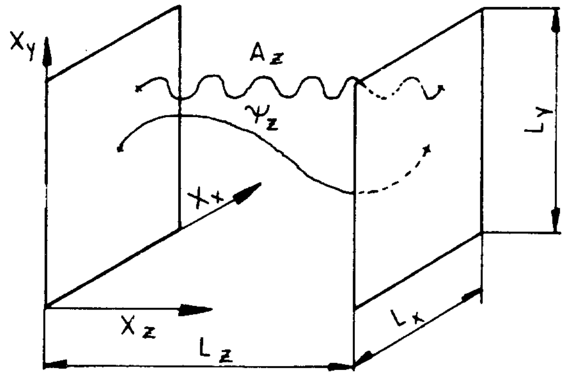

Fig. 1. Location of electron and photon fields.

was used in quantization. The relevant propagator vector for the photon is

$$\vec{k} = \begin{Bmatrix} \dfrac{2\pi}{L_x} n_x \\ \dfrac{2\pi}{L_y} n_y \\ \dfrac{2\pi}{L_z} n_z \end{Bmatrix}$$

n_x, n_y, and n_z have positive integer values. We use units such that $\hbar = c = 1$. The four-dimensional coordinates of the point are X_μ, where X_i constitute the space coordinates (X_x, X_y, X_z) and $X_0 = t$.

Because of the ultimate span L_z between the two plates, the summation in computing the propagator for the photon and electron and the calculation of the S-matrix must, unlike the hitherto performed computations, always and without exception be transacted in the direction of the component X_z.

Figure 2 shows the diagram relevant to the reaction.

The eigenvalues of the electron and photon field operators on the plates must equal zero. The relevant operator of the electron field is

$$\begin{aligned} \psi(x) &= V^{-1/2} \sum_{p_i} \left[\sum_{r=1,2} a^{(r)} u^{(r)} \sin(p_z X_z)\, e^{+ip_T X_T} \right. \\ &\quad \left. + \sum_{r=1,2} a^{(r)+} u^{(r)} \sin(p_z X_z)\, e^{-ip_T X_T} \right] \\ \bar{\psi}(x) &= V^{-1/2} \sum_{p_i} \left[\sum_{r=1,2} a^{(r)+} \bar{u}^{(r)} \sin(p_z X_z)\, e^{-ip_T X_T} \right. \\ &\quad \left. + \sum_{r=1,2} a^{(r)} \bar{u}^{(r)} \sin(p_z X_z)\, e^{ip_T X_T} \right] \end{aligned} \tag{1}$$

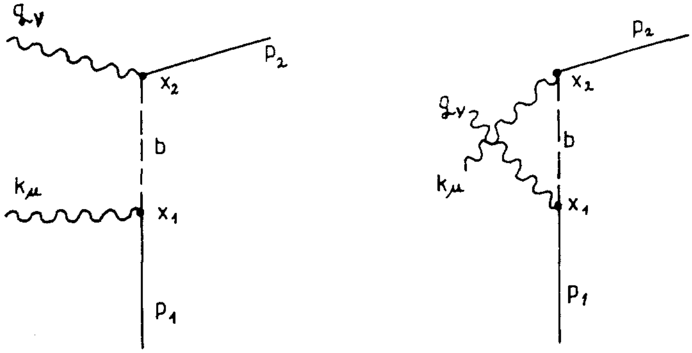

Fig. 2. Reaction diagrams.

In the field operators and S-matrix, the components X_z, p_z, q_z, k_z are parallel to the X_z coordinate, whereas X_T, p_T, k_T, q_T are the other three components of a four-vector.

In the operator for the electron field, a and a^+ constitute both the annihilation and the creation operators; u stands for the spinor; γ_μ, γ_ν are Dirac matrices. The relations given below must be fulfilled:

$$\bar{u} = u^* \gamma_4, \qquad \tfrac{1}{2}(\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu) = \vartheta_{\mu\nu} \tag{2}$$

$$\{\psi_\alpha(x_1), \bar{\psi}(x_2)\} \equiv i S_{\alpha\beta}(x_1 - x_2) \tag{3}$$

$$\{a_\alpha(p_i), a^+_\beta(p'_i)\} = \vartheta_{\alpha\beta} \vartheta_{p_i, p'_i} \tag{4}$$

$$\{a_\alpha(p_i), a_\beta(p'_i)\} = \{a^+_\alpha(p_i), a^+_\beta(p_i)\} = 0 \tag{5}$$

The propagator for the electron is computed by means of relation (3) in the interaction representation. A computation will furnish a propagator in our case:

$$S_F = \frac{i}{4} \lim_{\varepsilon \to 0} \frac{1}{(2\pi)^3} \int d^3 p_T \sum_{p_z} e^{ip(x-y)} \frac{i\gamma p - m}{p^2 + m^2 - i\varepsilon} \tag{6}$$

In our example the usual connection was not applied:

$$\sum_{P_i} \to \frac{V}{(2\pi)^3} \int \int \int d^3 P$$

When computing the propagator for the electron, L_z cannot tend toward infinity in the direction of the axis z because the two plates are spaced by a finite span L_z. For this reason it is imperative to conscientiously carry out the summation in the direction of the axis z. This is where this method differs from the Feynman propagators normally used.

Therefore we have to use the link

$$\sum_{p_i} \to \frac{V}{(2\pi)^2} \int\int \sum_{P_z} dp_x\, dp_y$$

The relevant operator of the photon field is

$$A_\mu(x) = \left(\frac{1}{2k_0 V}\right)^{1/2} \sum_{k_i} [a e_\mu^{(\lambda)} \sin(k_z X_z)\, e^{ik_T X_T} + a^+ e_\mu^{(\lambda)} \sin(k_z X_z)\, e^{-ik_T X_T}] \quad (7)$$

$$A_\nu(x) = \left(\frac{1}{2g_0}\right)^{1/2} \sum_{g_i} [a e_\nu^{(\lambda)} \sin(g_z X_z)\, e^{ig_T X_T} + a^+ e_\nu^{(\lambda)} \sin(g_z X_z)\, e^{-ig_T X_T}] \quad (8)$$

$$k_0 = |k_i|, \qquad g_0 = |g_i|$$

In the operators, a and a^+ constitute the relevant annihilation and creation operators; $e_\mu^{(\lambda)}$ and $e_\nu^{(\lambda)}$ constitute the corresponding polarization vectors. The relations given below must be satisfied:

$$\begin{aligned} &[a_\mu(k_i), a_\nu^+(k_i')] = \vartheta_{\mu\nu}\vartheta_{k_i,k_i'} \\ &[a_\mu(k_i), a_\nu(k_i')] = [a_\mu^+(k_i), a_\nu^+(k_i')] = 0 \end{aligned} \quad (9)$$

3. COMPUTATION OF THE S-MATRIX

The vertex diagrams for our case are depicted in Figure 2. The relevant S-matrix of the lowest order is

$$\begin{aligned} &\langle p_2, g|S|p_1, k\rangle \\ &= e_0^2 \int d^3X_{T_1} \int_0^{L_z} dX_{z1} \int d^3X_{T_2} \int_0^{L_z} dX_{z2} \langle p_2|\bar{\psi}|0\rangle \\ &\times \gamma_\mu S_F(x_1 - x_2)\gamma_\nu \langle 0|\psi(x_2)|p_1\rangle (:\langle g|A_\mu(x_1)A_\nu(x_2)|k\rangle:) \end{aligned} \quad (10)$$

The S-matrix (10) equals the S-matrix for Compton scattering. When computing the S-matrix we may encounter serious difficulty in the attempt to carry out integration along the "vertexes." The case quoted above features the span L_z in the direction of component X_z with the two plates pulled apart, which renders integration from minus to plus infinity virtually impossible. In the direction of component X_z, integration extends from zero to L_z, and in the direction of the other three components, from minus to plus infinity. Having inserted operators (1), (7), and (8) in the S-matrix (10) and carrying out the calculation, we obtain

$$\langle p_2, g|S|p_1, k\rangle$$
$$= V^{-2}\frac{ie_0^2}{(4k_0g_0)^{1/2}}\int d^3b_T\sum_{b_z}\bar{u}_2(p_2)\gamma_\mu e_\mu^{(\lambda_2)}\frac{1}{(2\pi)^3}\frac{i\gamma_T b_T + i\gamma_z b_z - m}{(b_T^2+b_z^2+m^2)b_z^2}$$
$$\times \sin^2\left(b_z\frac{L_z}{2}\right) e_\nu^{(\lambda_1)}\gamma_\nu u(p_1)(2\pi)^6\vartheta^3(-p_{2T}+b_T-g_T)$$
$$\times \vartheta^3(-b_T+p_{1T}+k_T)\, e^{-ip_{2z}L_z/2}\frac{\sin(p_{2z}L_z/2)}{p_{2z}}\, e^{ip_{1z}L_z/2}\frac{\sin(p_{1z}L_z/2)}{p_{1z}}$$
$$\times e^{-ig_zL_z/2}\frac{\sin(g_zL_z/2)}{g_z}\, e^{ik_zL_z/2}\frac{\sin(k_zL_z/2)}{k_z}+(k\rightleftharpoons g,\, \lambda_1\rightleftharpoons\lambda_2) \qquad (11)$$

The next problem when computing the S-matrix is the inability to integrate according to the momentum in the propagator, which necessitates summation strictly along component X_z,

$$b_z = \frac{2\pi}{L_z}n_z$$

The value for b_z is now inserted in the S-matrix and summed according to n_z from one to infinity. As regards the other components, integration is performed according to the usual procedures. An elaborate computation yields the result

$$\langle p_2, g|S|p_1, k\rangle$$
$$= (L_y, L_x)^{-2}\frac{1}{(4k_0g_0)^{1/2}}\bar{u}_2(p_2)\gamma_\mu\, e_\mu^{(\lambda_2)}$$
$$\times\left(\frac{i\gamma_\mu(p_1+k)-m}{(p_1+k)^2+m^2}-\frac{i\gamma_z}{4L_z[(p_{1T}+k_T)^2+m^2]^{1/2}[(p_{1T}+k_T)^2+m^2]}\right.$$
$$+\frac{i^2\gamma_z[(p_{1T}+k_T)^2+m^2]^{1/2}-[i\gamma_T(p_{1T}+k_T)-m]}{i[(p_{1T}+k_T)^2+m^2]^{1/2}[(p_{1T}+k_T)^2+m^2]}$$
$$\times\frac{\pi}{2L_z^2}\coth\{[(p_{1T}+k_T)^2+m^2]^{1/2}L_z\}$$
$$+\frac{2[i\gamma_T(p_{1T}+k_T)-m]+i^2\gamma_z[(\rho_{1T}+k_T)^2+m^2]^{1/2}}{4i[(p_{1T}+k_T)^2+m^2]^{1/2}[(p_{1T}+k_T)^2+m^2]}$$
$$\times\frac{\pi}{2L_z^2}\coth\left\{[(p_{1T}+k_T)^2+m^2]^{1/2}\frac{L_z}{2}\right\}$$

$$+\frac{i\gamma_u(p_1-g)-m}{(p_1-g)^2+m^2}-\frac{i\gamma_z}{4L_z^2[(p_{1T}-g_T)^2+m^2]^{1/2}[(p_{1T}-g_T)^2+m^2]}$$

$$+\frac{i^2\gamma_z[(p_{1T}-g_T)^2+m^2]^{1/2}-[i\gamma_T(p_{1T}-g_T)-m]}{i[(p_{1T}-g_T)^2+m^2][(p_{1T}-g_T)^2+m^2]^{1/2}}$$

$$\times\frac{\pi}{2L_z^2}\coth\left\{[(p_{1T}-g_T)^2+m^2]^{1/2}L_z\}\right\}$$

$$+\frac{2[i\gamma_T(p_{1T}-g_T)-m]-i^2\gamma_z[(p_{1T}-g_T)^2+m^2]^{1/2}}{4i[(p_{1T}-g_T)^2+m^2][(p_{1T}-g_T)^2+m^2]^{1/2}}$$

$$\times\frac{\pi}{2L_z^2}\coth\left\{[(p_{1T}-g_T)^2+m^2]^{1/2}\frac{L_z}{2}\right\}\Bigg)$$

$$e_\nu^{(\lambda_1)}\gamma_\nu u_1(p_1)2\pi^3\vartheta^3(p_{1T}+k_T-p_{2T}-g_T)\vartheta(n_{p_{1z}}+n_{k_z}-n_{p_{2z}}-ng_z)$$

$$=2\pi^3\vartheta^3(p_{1T}+k_T-p_{2T}-g_T)\vartheta(n_{p_{1z}}+n_{kz}-n_{p_{2z}}-n_{g_z})F \qquad (12)$$

The terms in the S-matrix containing L_z and hyperbolic functions constitute a correction due to the "vacuum" field between the two plates. The smaller the span between the plates, the greater the impact of the vacuum field.

With the plates being pushed apart the impact of the vacuum field diminishes and when $L_z=\infty$ all the corrective terms within the limit equal zero and as a result we obtain the well-known S-matrix for photon–electron or Compton scattering.

4. COMPUTATION OF THE CROSS SECTION

Proceeding with the computation of the S-matrix and the scattering cross section, a laboratory system (with the electron in the initial state at rest) is assumed.

The cross section is computed as shown in the equation below (Gupta, 1977):

$$\sigma=\frac{1}{(2\pi)^2|p_{1i}/p_{10}-k_i/k_0|}\int d\Omega\frac{|p_{2i}|p_0}{\rho(p_{10}+g_0)/\rho p_{10}}\bar{\Sigma}|F(p_2,g;p_1,k)|^2 \qquad (13)$$

$$\bar{\Sigma}=\bar{\Sigma}_{\text{spin}}\bar{\Sigma}_{\text{pol}}$$

$\bar{\Sigma}_{\text{spin}}$ denotes the average over the initial spin states and the summation over the final spin states of the electron. It should be pointed out though that this is true in instances when the spin states of the particle during the process of collision are of no interest. $\bar{\Sigma}_{\text{pol}}$ is of similar significance for the average and the summation for the polarization states of photon.

To simplify the S-matrix, we use the following notations:

$$p_{1i}=0, \qquad p_{10}=m$$

$$k_i=p_{2i}+q_i, \qquad p_1^2=p_2^2=-m^2, \qquad k^2=q^2=0$$

$$(p_1+k)^2+m^2=2p_1k, \qquad (p_1-q)^2+m^2=-2p_1q$$

$$(p_{1T}+k_T)^2+m^2=2p_{1T}k_T, \qquad (p_{1T}-q_T)^2+m^2=-p_{1T}q_T$$

For transverse photons, the polarization vector e does not contain the fourth component, which makes further simplification possible:

$$(ip_1\gamma-m)(e_i^{(\lambda_2)}\gamma_i)u(p_1)=0, \qquad (ip_1\gamma-m)(e_i^{(\lambda_1)}\gamma_i)u(p)=0$$

Having first simplified the S-matrix (12) by means of the equations quoted above, we now compute F^2. We then insert F^2 into equation (13) and compute the scattering cross section for polarized light:

$$\sigma_{\text{pol}}=\tfrac{1}{4}r_0^2\int d\Omega\Bigg(\frac{g_0^2}{k_0^2}\left[\frac{k_0}{g_0}+\frac{g_0}{k_0}-2+(ee')^2\right]$$
$$-\frac{p_{20}g_0^3}{4k_0g_0m}\left\{-\frac{1}{\pi^2 16L_z^4k_0^3m}-\frac{1}{\pi^2 16L_z^4g_0^3m}-\frac{1}{\pi 8(k_0g_0)(k_0g_0)^{1/2}mL_z^4}\right.$$
$$\left.+\frac{\pi^2}{2L_z^4k_0^4}\coth^2[(2mk_0)^{1/2}L_z]+\frac{\pi^2}{2L_z^4g_0^4}\coth^2[(2mg_0)^{1/2}L_z]\right\}(ee')^2\Bigg) \tag{14}$$

In cases when L_z approaches infinity, the computed scattering cross section agrees with the Klein-Nishine expression. The corrective terms then all equal zero.

At this point we calculate the scattering cross section for the case when $k_0=q_0$ and $k_0\ll m_0$. In equation (14) we insert

$$k_0=\frac{2\pi}{\lambda}, \qquad q_0=\frac{2\pi}{\lambda'}, \qquad \lambda=\lambda'$$

A short calculation yields

$$\sigma_{\text{pol}}=\frac{1}{4}\int d\Omega\, r_0^2(ee')^2\left(1-\frac{1}{128\pi^2}\frac{\lambda^4}{L_z^4}\right) \tag{15}$$

In equation (15), λ stands for the wavelength of the photon. The condition $\lambda\leq L_z$ must be taken into account. For the summation of photon polarization states we apply the familiar relation

$$\bar{\Sigma}_{\text{pol}}(ee')^2 = \tfrac{1}{2}(1+\cos^2\theta) \qquad (16)$$

The scattering cross section of unpolarized light for the low-energy photon is

$$\sigma = r_0^2 \int d\Omega\,[(1-\tfrac{1}{2}\sin^2\theta)\left(1-\frac{1}{128\pi^2}\frac{\lambda^4}{L_z^4}\right) \qquad (17)$$

From equations (14), (15), and (17) it is evident that the influence of the correction terms of the vacuum field upon the scattering cross section is the greater, the smaller the span L_z between the two plates. In instances when the photon wavelength is significantly shorter that the span between the two plates, the correction term of the vacuum field may be omitted.

All correction terms of the vacuum field are proportional to L_z. The dependence of the attractive forces upon L_z in uncharged conductive plates was worked out by Casimir. Also dependent upon L_z is the vacuum energy density between the two plates. When the L_z span between the plates approaches infinity, all correction terms of the vacuum field as well as the vacuum energy equal zero. Equations (14), (15), and (17) for the scattering cross sections when the L_z span between the plates approaches infinity are equivalent to the values computed so far. The computations carried out for the S-matrix and the scattering cross section for the reactions between elementary particles, e.g., photon–electron scattering, positron–electron annihilation, bremsstrahlung, and the like, do not take the influence of the vacuum field into account. The computation of the S-matrix and the scattering cross section for the photon–electron scattering briefly presented here illustrates the vacuum field influence. The purpose of the theoretical analysis of the reaction is to find out by a concrete example the influence of the vacuum field upon the photon–electron scattering. To ascertain the correctness of the equations given above, experiments must be carried out.

In order to verify equations (14), (15), and (17) experimentally, we intend to perform measurements of the scattering of light of various wavelengths (0.1–10 kÅ) on electrons located between two plates with a span of 0.1–1 μm. The span will thus be equal to the one Sparnaay obtained in gauging the Casimir effect.

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